THREE-PHASE CIRCUITS
CHAPTER 12

12.1 What is a Three-Phase Circuit?
12.2 Balanced Three-Phase Voltages
12.3 Balanced Three-Phase Connections
12.4 Power in a Balanced System
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12.1 Advantages of Three-Phase Circuit?

1. **Most** of the electric power is generated and distributed in three-phase.
2. The instantaneous power in a three-phase system is **constant** (rather than pulsating).
3. The amount of power generated by a three-phase generator is more **economical** than that of a single-phase generator.
4. The amount of wire required for a three-phase system is **less than** that required for an equivalent single-phase system.
Schematic Diagram of a Power System

Color Key:
Black: Generation
Blue: Transmission
Green: Distribution

Generating Station

Transmission lines
765, 500, 345, 230, and 138 kV

Transmission Customer
138kV or 230kV

Substation Step Down Transformer

Subtransmission Customer
26kV and 69kV

Primary Customer
13kV and 4kV

Secondary Customer
120V and 240V
US Electric Energy Generation Mix

- Coal: 48%
- Natural Gas: 21%
- Nuclear: 20%
- Hydroelectric: 6%
- Other Gases: 0.3%
- Petroleum: 1%
- Other Renewables: 3%
- Other: 0.3%
Thermal Power Plants
Natural Gas Power Plants
Hydro Power Plants
Renewable Power Plants
12.2 Balanced Three-Phase Voltages

- A three-phase generator consists of a rotating magnet (in the rotor) surrounded by stationary windings (in the stator).

A three-phase generator

The generated voltages
12.1 Definition of a Three-Phase Circuit

- It is a system with a generator consisting of **three sources** having the same amplitude and frequency but **out of phase** with each other by 120°.
12.2 Balanced Three-Phase Voltages

- Two possible configurations:

Three-phase voltage sources: (a) Y-connected ; (b) Δ-connected
12.2 Balance Three-Phase Voltages

- **Balanced phase voltages** are equal in magnitude and are out of phase with each other by 120°.

- The **phase sequence** is the time order in which the voltages pass through their respective maximum values.
  - a-b-c (or positive sequence)
  - a-c-b (or negative sequence)

- A **balanced load** is one in which the phase impedances are equal in magnitude and in phase.
**12.2 Balance Three-Phase Voltages**

**Example 1:** Determine the phase sequence of the set of voltages, and the corresponding line voltages

\[
\begin{align*}
v_{an} &= 200 \cos(\omega t + 10^\circ) \\
v_{bn} &= 200 \cos(\omega t - 230^\circ) \\
v_{cn} &= 200 \cos(\omega t - 110^\circ)
\end{align*}
\]

**Answer:** The voltages can be expressed in phasor form as

\[
\begin{align*}
V_{an} &= 200 \angle 10^\circ \text{ V} \\
V_{bn} &= 200 \angle 130^\circ \text{ V} \\
V_{cn} &= 200 \angle -110^\circ \text{ V}
\end{align*}
\]

\[
\begin{align*}
V_{ab} &= 200\sqrt{3} \angle -20^\circ \text{ V} \\
V_{bc} &= 200\sqrt{3} \angle 100^\circ \text{ V} \\
V_{ca} &= 200\sqrt{3} \angle -140^\circ \text{ V}
\end{align*}
\]

\(V_{an}\) leads \(V_{cn}\) by 120° and \(V_{cn}\) in turn leads \(V_{bn}\) by 120°. Hence, we have an **a-c-b** sequence (or negative sequence).
12.3 Balanced Three-Phase Connections

- Four possible connections
  1. Y-Y connection
  2. Y-Δ connection
  3. Δ-Δ connection
  4. Δ-Y connection
A balanced Y-Y system is a three-phase system with a balanced Y-connected source and a balanced Y-connected load.

\[ I_n = -(I_a + I_b + I_c) = 0 \]
12.3 Balanced Three-Phase Connection

Example 2:
Obtain the line currents

Answer:

Using the per-phase circuit shown above,

\[ I_a = \frac{440 \angle 0^\circ}{6 - j8} = 44 \angle 53.13^\circ \text{ A} \]

\[ I_b = I_a \angle -120^\circ = 44 \angle -66.87^\circ \text{ A} \]

\[ I_c = I_a \angle 120^\circ = 44 \angle 173.13^\circ \text{ A} \]
12.3 Balanced Three-Phase Connection

A balanced Y-Δ system is a three-phase system with a balanced Y-connected source and a balanced Δ-connected load.
12.3 Balanced Three-Phase Connection

Example 3:

Solve for the line currents in the Y-Δ circuit of Fig. 12.45. Take $Z_\Delta = 60 \angle 45^\circ \Omega$.

**Answer:**

$$Z_Y = \frac{Z_\Delta}{3} = 20 \angle 45^\circ \Omega$$

$$I_a = \frac{110 \angle 0^\circ}{20 \angle 45^\circ} = 5.5 \angle -45^\circ \ A$$

$$I_b = I_a \angle -120^\circ = 5.5 \angle -165^\circ \ A$$

$$I_c = I_a \angle 120^\circ = 5.5 \angle 75^\circ \ A$$

**Figure 12.45**
A balanced $\Delta$-$\Delta$ system is a three-phase system with a balanced $\Delta$-connected source and a balanced $\Delta$-connected load.
Example 4:

Refer to the $\Delta - \Delta$ circuit in Fig. 12.51. Find the line and phase currents. Assume that the load impedance is $Z_L = 12 + j9 \ \Omega$ per phase.

Answer:

$$Z_\Delta = 12 + j9 = 15 \angle 36.87^\circ$$

The phase currents are

$$I_{AB} = \frac{210 \angle 0^\circ}{15 \angle 36.87^\circ} = 14 \angle -36.87^\circ \ A$$
$$I_{BC} = I_{AB} \angle -120^\circ = 14 \angle -156.87^\circ \ A$$
$$I_{CA} = I_{AB} \angle 120^\circ = 14 \angle 83.13^\circ \ A$$

The line currents are

$$I_a = I_{AB} \sqrt{3} \angle -30^\circ = 24.25 \angle -66.87^\circ \ A$$
$$I_b = I_a \angle -120^\circ = 24.25 \angle -186.87^\circ \ A$$
$$I_c = I_a \angle 120^\circ = 24.25 \angle 53.13^\circ \ A$$
A balanced $\Delta$-Y system is a three-phase system with a balanced y-connected source and a balanced y-connected load.
Example 5: let $V_{ab} = 440 \angle 10^\circ$, $V_{bc} = 440 \angle 250^\circ$, $V_{ca} = 440 \angle 130^\circ$. Find the line currents.

Answer:

Convert the delta-connected source to an equivalent wye-connected source and consider the single-phase equivalent.

$$I_a = \frac{440 \angle (10^\circ - 30^\circ)}{\sqrt{3} Z_Y}$$

where

$$Z_Y = 3 + j2 + 10 - j8 = 13 - j6 = 14.32 \angle -24.78^\circ$$

$$I_a = \frac{440 \angle -20^\circ}{\sqrt{3} (14.32 \angle -24.78^\circ)} = 17.74 \angle 4.78^\circ \ A$$

$$I_b = I_a \angle -120^\circ = 17.74 \angle -115.22^\circ \ A$$

$$I_c = I_a \angle 120^\circ = 17.74 \angle 124.78^\circ \ A$$
12.4 Power in a Balanced System

\[ P = 3V_p I \cos(\theta) = \sqrt{3}V_L I \cos(\theta) \]

\[ Q = 3V_p I \sin(\theta) = \sqrt{3}V_L I \sin(\theta) \]

\[ S = 3V_p I = \sqrt{3}V_L I \]
12.3 Balanced Three-Phase Systems

Example 6
Determine the total complex power at the source and at the load

Answer:
At the source:
$S_s = (2087 + j834.6) \text{ VA}$

At the load:
$S_L = (1392 + j1113) \text{ VA}$
12.3 Balanced Load: Example 7

The following three parallel-connected three-phase loads are fed by a balanced three-phase source:
Load 1: 250 kVA, 0.8 pf lagging
Load 2: 300 kVA, 0.95 pf leading
Load 3: 450 kVA, unity pf

If the line voltage is 13.8 kV, calculate the line current and the power factor of the source. Assume that the line impedance is zero.

Answer:

\begin{align*}
\text{pf} &= 0.8 \quad \text{(lagging)} \quad \Rightarrow \quad \theta = \cos^{-1}(0.8) = 36.87^\circ \\
S_1 &= 250 \angle 36.87^\circ = 200 + j150 \text{ kVA} \\

\text{pf} &= 0.95 \quad \text{(leading)} \quad \Rightarrow \quad \theta = \cos^{-1}(0.95) = -18.19^\circ \\
S_2 &= 300 \angle -18.19^\circ = 285 - j93.65 \text{ kVA} \\

\text{pf} &= 1.0 \quad \Rightarrow \quad \theta = \cos^{-1}(1) = 0^\circ \\
S_3 &= 450 \text{ kVA} \\

S_T &= S_1 + S_2 + S_3 = 935 + j56.35 = 936.7 \angle 3.45^\circ \text{ kVA} \\
|S_T| &= \sqrt{3} V_L I_L \\
I_L &= \frac{936.7 \times 10^3}{\sqrt{3} (13.8 \times 10^3)} = 39.19 \text{ A rms} \\
\text{pf} &= \cos \theta = \cos(3.45^\circ) = 0.9982 \quad \text{(lagging)}
\end{align*}
12.5 Unbalanced 4-Wire Circuit

An unbalanced system is due to unbalanced voltage sources or an unbalanced load.

- The total power is not simply three times the power in one phase, but the sum of the powers in the three phases.
- To calculate power in an unbalanced three-phase system requires that we find the power in each phase.

\[
I_a = \frac{V_{AN}}{Z_A}, \quad I_b = \frac{V_{BN}}{Z_B}, \quad I_c = \frac{V_{CN}}{Z_C},
\]

\[
I_n = -(I_a + I_b + I_c)
\]
12.5 Unbalanced Three-Wire Y-Circuit

Determine the line currents for the three-phase circuit of Fig. 12.64. Let \( V_a = 110 \angle 0^\circ \), \( V_b = 110 \angle -120^\circ \), \( V_c = 110 \angle 120^\circ \) V.

Answer (mesh analysis):

\[
(100 + j80)I_1 - (20 + j30)I_2 = V_a - V_b = 165 + j95.263 \quad (1)
\]

\[-(20 + j30)I_1 + (80 - j10)I_2 = V_b - V_c = -j190.53 \quad (2)
\]

Solving (1) and (2) gives \( I_1 = 1.8616 - j0.6084 \), \( I_2 = 0.9088 - j1.722 \).

\[
I_a = I_1 = 1.9585 \angle -18.1^\circ \text{ A}, \quad I_b = I_2 - I_1 = -0.528 - j1.1136 = 1.4656 \angle -130.55^\circ \text{ A}
\]

\[
I_c = -I_2 = 1.947 \angle 117.8^\circ \text{ A}
\]

Verify the answer using nodal analysis.
12.5 Unbalanced Three-Wire Δ-Circuit

Calculate the line currents and total real and reactive power supplied by the source:

Answer:

\[ I_3 = \frac{(440)(-1.5 + j0.866)}{j5} = 152.42 \angle 60^\circ \]

\[ I_1 = I_3 + 76.21 \angle -60^\circ = 114.315 + j66 = 132 \angle 30^\circ \]

\[ I_2 = I_3 - j38.1 = 76.21 + j93.9 = 120.93 \angle 50.94^\circ \]

\[ I_a = I_1 = 132 \angle 30^\circ \text{ A} \]

\[ I_b = I_2 - I_1 = -38.105 + j27.9 = 47.23 \angle 143.8^\circ \text{ A} \]

\[ I_c = -I_2 = 120.9 \angle 230.9^\circ \text{ A} \]

Rework the problem using a simpler method (i.e., source conversion)
Three Watt-Meter Method: $P_T = P_1 + P_2 + P_3$

Two Watt-Meter Method: $P_T = P_1 + P_2$

**Special case:** for a balanced load, the total reactive power may be computed from the readings of the two wattmeters:

$$Q_T = \sqrt{3}(P_2 - P_1)$$
The unbalanced load is supplied by a balanced source such that \( V_{ab} = 208\angle 0^\circ \) V with positive phase sequence. Calculate the reading of each wattmeter.

\[
\begin{align*}
I_{AB} &= \frac{V_{ab}}{Z_{Ab}} = \frac{208\angle 0^\circ}{20} = 10.4\angle 0^\circ \\
I_{BC} &= \frac{V_{bc}}{Z_{BC}} = \frac{208\angle -120^\circ}{10\sqrt{2}\angle -45^\circ} = 14.708\angle -75^\circ \\
I_{CA} &= \frac{V_{ca}}{Z_{CA}} = \frac{208\angle 120^\circ}{13\angle 22.62^\circ} = 16\angle 97.38^\circ \\
I_{aA} &= I_{AB} - I_{CA} = 10.4\angle 0^\circ - 16\angle 97.38^\circ \\
I_{aA} &= 10.4 + 2.055 - j15.867 \\
I_{aA} &= 20.171\angle -51.87^\circ \\
I_{cC} &= I_{CA} - I_{BC} = 16\angle 97.83^\circ - 14.708\angle -75^\circ \\
I_{cC} &= 30.64\angle 101.03^\circ
\end{align*}
\]

Verify the answer by computing the power consumed by each resistor.
12.6 Application: Residential Wiring (single-phase 3-wire)

A 120/240 household power system
12.6 Application – Residential Wiring

A typical wiring diagram of a room
12.6 Application – Residential Wiring

Single-phase three-wire residential wiring
Consider the single-phase three-wire system shown in Fig. 12.78. Find the current in the neutral wire and the complex power supplied by each source. Take $V_2$ as a $115 \angle 0^\circ \text{-V}$, 60-Hz source.

Answer:

\[
I_1 = \frac{\Delta_1}{\Delta} = \frac{115 \times 2082 \angle 18.47^\circ}{19214 \angle 48.09^\circ} = 12.52 \angle -29.62^\circ
\]

\[
I_2 = \frac{\Delta_2}{\Delta} = \frac{115 \times 1884.9 \angle 14.48^\circ}{19124 \angle 48.09^\circ} = 11.33 \angle -33.61^\circ
\]

\[
I_n = I_2 - I_1 = \frac{\Delta_2 - \Delta_1}{\Delta} = \frac{(115)(-150 - j188.5)}{12,775 + j14,231.75} = 1.448 \angle -176.6^\circ \text{ A}
\]

\[
S_1 = V_1 I_1^* = (115)(12.52 \angle 29.62^\circ) = 1252 + j711.6 \text{ VA}
\]

\[
S_2 = V_2 I_2^* = (115)(1.33 \angle 33.61^\circ) = 1085 + j721.2 \text{ VA}
\]
UTILITY APPLICATION OF CAPACITORS FOR VOLTAGE REGULATION AND POWER TRANSFER

Capacitors are placed in series with high Voltage transmission lines to improve voltage Regulation and power transfer capability.

Capacitor are also used in medium voltage power distribution lines to regulate voltage and reduce power losses.
ENERGY METER (KILO-WATT-HOUR METER)

Old electro-mechanical meter – records only energy consumed over a time period using mechanical dials.

New smart meter – records average power, reactive power, power factor, power demand, energy over a period of time, etc …
- Remote (wireless) reading
- Remote turn ON and OFF
- Communicates with the utility operator in near-continuous basis
- Able to communicate with home appliances such as HVAC thermostat….
Power Demand:

1 customer:

250,000 customers:
Electrical Safety:

- Electricity is an integral part of today's modern world, and sometimes it is easy to forget just how dangerous it can be. Given the correct circumstances, it can shock you painfully or even kill you.

- According to the Bureau of Labor Statistics Census of Fatal Occupational Injuries Research File, electrocution is the fifth leading cause of work-related deaths.

- The severity of injury from electrical shock depends on the amount of electrical current and the length of time the current passes through the body.
  - For example, 100 mA going through the body for just 2 seconds is enough to cause death.
  - The amount of internal current a person can withstand and still be able to control the muscles of the arm and hand can be less than 10 mA.
  - Currents above 10 mA can paralyze or “freeze” muscles. When this “freezing” happens, a person is no longer able to release a tool, wire, or other object. In fact, the electrified object may be held even more tightly, resulting in longer exposure to the shocking current.
# Effects of Electrical Current in the Human Body

<table>
<thead>
<tr>
<th>Current</th>
<th>Reaction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below 1 milliampere</td>
<td>Generally not perceptible.</td>
</tr>
<tr>
<td>1 milliampere</td>
<td>Faint tingle.</td>
</tr>
<tr>
<td>5 milliamperes</td>
<td>Slight shock felt; not painful but disturbing. Average individual can let go. Strong involuntary reactions can lead to other injuries.</td>
</tr>
<tr>
<td>6–25 milliamperes</td>
<td>Painful shock, loss of muscular control. The freezing current or &quot;let-go&quot; range. Individual cannot let go, but can be thrown away from the circuit if extensor muscles are stimulated.*</td>
</tr>
<tr>
<td>(women)</td>
<td></td>
</tr>
<tr>
<td>9–30 milliamperes</td>
<td></td>
</tr>
<tr>
<td>(men)</td>
<td></td>
</tr>
<tr>
<td>50–150 milliamperes</td>
<td>Extreme pain, respiratory arrest (breathing stops), severe muscular contractions. Death is possible.</td>
</tr>
<tr>
<td>1,000–4,300 milliamperes</td>
<td>Rhythmic pumping action of the heart ceases. Muscular contraction and nerve damage occur; death likely.</td>
</tr>
<tr>
<td>10,000 milliamperes</td>
<td>Cardiac arrest and severe burns occur. Death is probable.</td>
</tr>
<tr>
<td>15,000 milliamperes</td>
<td>Lowest overcurrent at which a typical fuse or circuit breaker opens a circuit!</td>
</tr>
</tbody>
</table>

*Never use a three-prong grounding plug with the ground prong broken off.