Diode Rectifiers

EE 442-642
Current continues to flow for a while even after the input voltage has gone negative.
Half Bridge Rectifier Circuit: Load with dc back-emf

- Current begins to flow when the input voltage exceeds the dc back-emf.
- Current continues to flows for a while even after the input voltage has gone below the dc back-emf.
The DC currents of the two half-wave rectifiers are equal and opposite, hence, there is no DC current for creating a transformer core saturation problem.

Each diode carries half of the load average current, but the same peak load current.

Note that the $V_{RRM}$ rating of the diodes must be chosen to be higher than $2V_m$ to avoid reverse breakdown.

The center-tap transformer is considered bulky with additional losses.
Full Bridge Rectifier – Simple R Load

Average value of output voltage: \[ V_{do} = (2\sqrt{2}/\pi)V_s \approx 0.9V_s = (2/\pi)V_m = 0.637V_m \]

where \( V_s \) and \( V_m \) are the RMS and peak values of input voltage.

Rectification Ratio = \( \frac{P_{dc}}{P_{ac}} = 0.81 \) or 81%

Form Factor (FF) of DC side voltage (or current) = \( \frac{V_{rms}}{V_{dc}} = 1.11 \)

Ripple Factor = rms value of AC component/DC component = \( (FF^2-1)^{1/2} = 0.48 \)

5-5
Filters

- Filters are employed in rectifier circuits for smoothing out the dc output voltage of the load.

- Ripple Reduction Factor at $f_r$:

\[
\frac{v_o}{v_L} = \frac{R}{\sqrt{R^2 + (2\pi f_r L_f)^2}}
\]

\[
\frac{v_o}{v_L} = \left| \frac{1}{1 - (2\pi f_r)^2 L_f C_f} \right|
\]

\[
\int \sin b_1 x \sin b_2 x \, dx = \frac{\sin((b_2 - b_1)x)}{2(b_2 - b_1)} - \frac{\sin((b_1 + b_2)x)}{2(b_1 + b_2)} + C \quad \text{(for } |b_1| \neq |b_2|)\]

\[
\int \sin a_1 x \cos a_2 x \, dx = -\frac{\cos((a_1 - a_2)x)}{2(a_1 - a_2)} - \frac{\cos((a_1 + a_2)x)}{2(a_1 + a_2)} + C \quad \text{(for } |a_1| \neq |a_2|)\]

5-6
Inductor DC Filter

- Minimum value of inductance required to maintain a continuous current is known as the critical inductance $L_C$:

  \[ L_C = \frac{R}{6\pi f_i} \]  

  (Full-wave)

- The choice of the input inductance depends on the required ripple factor. Ripple voltage of a rectifier without filtering:

  \[ v_{L_{n}} = \frac{-4V_m}{\pi (n^2 - 1)} \]

  \[ RF = \sqrt{2 \sum_{n=2,4,8} \left( \frac{1}{n^2 - 1} \right)^2} \]

- Considering only the lowest-order harmonic ($n = 2$), the output ripple factor of a simple inductor-input dc filter is

  \[ \text{Filtered} \quad RF = \frac{0.4714}{\sqrt{1 + (4\pi f_i L_f / R)^2}} \]
Full Bridge Rectifier with dc-side Voltage

Average value of DC-side current (obtained numerically):

Zero current corresponds to dc voltage equal to the peak of the input ac voltage

\[
\frac{V_d}{V_{dc}(= 0.9 V_e)} = \frac{\sqrt{2}}{0.9} = 1.57
\]
Full Bridge Rectifier – Simple Constant Load Current

RSM value of source current
\[ I_s = I_d \]

RMS value of fundamental current
\[ I_{s1} = (2\sqrt{2} / \pi)I_d \approx 0.9I_d \]

RMS value of harmonic current
\[ I_{sh} = I_{s1}/h, \quad h = 3, 5, 7, \ldots \]

Current THD
\[ THD = 100[\sqrt{(\pi^2 / 8) - 1}] = 48.43\% \]

Displacement Power Factor
\[ DPF = 1 \]

Power Factor
\[ PF = 0.9 \]
Full Bridge Rectifier – Simple Constant Load Current (AC-side filtering)

RMS value of harmonic source current

\[ I_{sn} = \frac{1}{\left|\frac{1}{1 - (2n\pi f_i)^2 L_i C_i}\right|} I_m \]

THD of Source Current:

\[ \text{THD} = \sqrt{\sum_{n=3,5} \frac{1}{n^2} \left|\frac{1}{1 - (2n\pi f_i)^2 L_i C_i}\right|^2} \]
Diode-Rectifier with a Capacitor Filter

- Approximate value of peak-to-peak ripple:
  \[ V_{r(pp)} = \frac{V_m}{f \cdot R \cdot C} \]

- Average output voltage:
  \[ V_{dc} = V_m \left(1 - \frac{1}{2f \cdot R \cdot C}\right) \]

- RMS value of output ripple voltage:
  \[ V_{ac} = \frac{V_m}{2\sqrt{2f \cdot R \cdot C}} \]

- Ripple Factor:
  \[ RF = \frac{1}{\sqrt{2(2f \cdot R \cdot C - 1)}} \]

- Inrush resistance \( R_{\text{inrush}} \) is sometimes needed to limit the initial inrush current to a value below that of the diodes. It is usually placed on the DC side, then shorted out afterwards.

- In many cases, \( R_{\text{inrush}} \) is not needed if the Equivalent Series Resistance of the capacitor and cable/transformer wire resistance are sufficiently large.
Diode-Rectifier Bridge with AC-Side Inductance

Commutation angle: \[ \cos \mu = 1 - \frac{2\omega L_s I_d}{\sqrt{2}V_s} \]

Average of DC-side voltage: \[ V_d = 0.9V_s - \frac{2\omega L_s I_d}{\pi} \]
Voltage Distortion at PCC

- PCC is the point of common coupling
- Distorted current flow results in distorted voltage
Dual Voltage Rectifier

- In 115-V position, one capacitor at-a-time is charged from the input.
Three-Phase, Four-Wire System

- A common neutral wire is assumed
- The current in the neutral wire is composed mainly of the third harmonic and can be higher than the phase currents
3-Phase Rectifier Circuit

- Star Connection: direct currents in the secondary windings cause transformer core saturation problem.

- The transformer core saturation problem in the three-phase star rectifier can be avoided by zig-zag (or inter-star) connection.

- Three-phase Bridge (or 6-pulse) Rectifier – most commonly used in industry applications.
Three-Phase Bridge Rectifier with R Load

- Average value of DC-side voltage:
  \[ V_{do} = \frac{3}{\pi} \sqrt{3}V_m = 1.654V_m \]
  where \( V_m \) is the peak value of the phase voltage.

- RMS value of DC-side voltage:
  \[ V_{rms} = V_m \sqrt{\frac{3}{2} + \frac{9\sqrt{3}}{4\pi}} = 1.655V_m \]
Six-Phase Bridge Rectifier (12-Pulse)

- Average and RMS values of DC-side voltage:
  \[ V_{do} \approx V_{rms} \approx 1.712V_m \]

  where \( V_m \) is the peak value of the phase voltage.
Three-Phase Full-Bridge Rectifier with DC Current
Three-Phase, Full-Bridge Rectifier with DC Current

RSM value of source current
RMS value of fundamental current
RMS value of harmonic current
Current THD
Displacement Power Factor
Power Factor

\[ I_s = \sqrt{\frac{2}{3}} I_d = 0.816 I_d \]
\[ I_{s1} = \left(\sqrt{\frac{6}{\pi}}\right) I_d \approx 0.78 I_d \]
\[ I_{sh} = I_{s1} / h, \quad h = 3, 5, 7, \ldots \]
\[ THD = 100\left[\sqrt{\left(\frac{\pi^2}{9}\right)} - 1\right] = 31\% \]
\[ DPF = 1 \]
\[ PF = \frac{3}{\pi} = 0.955 \]
3-Phase, Full-Bridge Rectifier with Inductance

Commutation angle: \[ \cos \mu = 1 - \frac{2\omega L_s I_d}{\sqrt{2}V_{LL}} \]

Average of DC-side voltage: \[ V_d = 1.35V_{LL} - \frac{3\omega L_s I_d}{\pi} \]
3-Phase Rectifier with DC Source

(a) Circuit diagram of a 3-phase rectifier with a DC source. The diagram shows the phase sequence and the direction of the current $i_a$.

(b) Equivalent circuit of the rectifier showing the inductors $L_a$, $L_b$, and $L_c$, and the diodes $D_1$, $D_2$, $D_3$, and $D_4$.

(c) Waveforms showing the voltage $v_{an}$, current $i_a$, and the phase voltages $v_{ab}$, $v_{ac}$, and $v_{in}$. The waveform $v_{an}$ represents the output voltage, which is a DC level with ripples.
Three-Phase Rectifier with Capacitor Filter

- PSpice-based analysis