Power System Representations: Voltage-Current Relations

EE 340
One-line diagram
(simple power system)

- Machine ratings, impedances, consumed and/or supplied powers are usually included in the diagrams
Per-unit equivalent circuit

• Real power systems are convenient to analyze using their per-phase (since the system is three-phase) per-unit (since there are many transformers) equivalent circuits.

• Recall: given the base apparent power (3—phase) and base voltage (line-to-line), the base current and base impedance are given by

\[
I_{\text{base}} = \frac{S_{3\phi, \text{base}}}{\sqrt{3}V_{\text{LL, base}}} \\
Z_{\text{base}} = \frac{V_{\text{LL, base}}}{\sqrt{3}I_{\text{base}}} = \left(\frac{V_{\text{LL, base}}}{S_{3\phi, \text{base}}}\right)^2
\]
Per-unit system

• The base apparent power and base voltage are specified at a point in the circuit, and the other values are calculated from them.

• The base voltage varies by the voltage ratio of each transformer in the circuit but the base apparent power stays the same through the circuit.

• The per-unit impedance may be transformed from one base to another as

\[
Per-unit \ Z_{new} = per-unit \ Z_{old} \left( \frac{V_{old}}{V_{new}} \right)^2 \left( \frac{S_{new}}{S_{old}} \right)
\]
Example 10.2: a power system consists of one synchronous generator and one synchronous motor connected by two transformers and a transmission line. Create a per-phase, per-unit equivalent circuit of this power system using a base apparent power of 100 MVA and a base line voltage of the generator $G_1$ of 13.8 kV. Given that:

$G_1$ ratings: 100 MVA, 13.8 kV, $R = 0.1$ pu, $X_s = 0.9$ pu;
$T_1$ ratings: 100 MVA, 13.8/110 kV, $R = 0.01$ pu, $X_s = 0.05$ pu;
$T_2$ ratings: 50 MVA, 120/14.4 kV, $R = 0.01$ pu, $X_s = 0.05$ pu;
$M$ ratings: 50 MVA, 13.8 kV, $R = 0.1$ pu, $X_s = 1.1$ pu;
$L_1$ impedance: $R = 15$ Ω, $X = 75$ Ω.

\[
V_{\text{base},1} = 13.8 \text{ kV} \quad V_{\text{base},2} = V_{\text{base},1} \frac{110}{13.8} = 110 \text{ kV} \quad V_{\text{base},3} = V_{\text{base},2} \frac{14.4}{120} = 13.2 \text{ kV}
\]
Example (cont.)

\[
Z_{base,1} = \frac{V_{LL,base}^2}{S_{3\phi,base}} = \frac{(13.8 \text{kV})^2}{100 \text{ MVA}} = 1.904 \Omega
\]

\[
Z_{base,2} = \frac{V_{LL,base}^2}{S_{3\phi,base}} = \frac{(110 \text{kV})^2}{100 \text{ MVA}} = 121 \Omega
\]

\[
Z_{base,3} = \frac{V_{LL,base}^2}{S_{3\phi,base}} = \frac{(13.2 \text{kV})^2}{100 \text{ MVA}} = 1.743 \Omega
\]

\[
R_{G1,pu} = 0.1 \text{ per unit}
X_{G1,pu} = 0.9 \text{ per unit}
R_{T1,pu} = 0.01 \text{ per unit}
X_{T1,pu} = 0.05 \text{ per unit}
\]

\[
R_{line,system} = \frac{15}{121} = 0.124 \text{ per unit}
X_{line,system} = \frac{75}{121} = 0.620 \text{ per unit}
\]

\[
R_{M2,pu} = 0.1(14.8/13.2)^2 (100/50) = 0.219 \text{ per unit}
R_{T2,pu} = 0.01(14.4/13.2)^2 (100/50) = 0.238 \text{ per unit}
X_{M2,pu} = 1.1(14.8/13.2)^2 (100/50) = 2.405 \text{ per unit}
X_{T2,pu} = 0.05(14.4/13.2)^2 (100/50) = 0.119 \text{ per unit}
\]
Node equations

- Once the per-unit equivalent circuit is created, it can be used to find the voltages, currents, and powers at various points.
- The most common technique used to solve such circuits is nodal analysis. To simplify the equations,
  - Replace the generators by their Norton equivalent circuits
  - Replace the impedances by their equivalent admittances
  - Represent the loads by the current they draw (for now)
Node equations

• According to Kirchhoff’s current flow law (KCL), **the sum of all currents entering any node equals to the sum of all currents leaving the node.**
• KCL can be used to establish and solve a system of simultaneous equations with the unknown node voltages.
• Assuming that the current from the current sources are entering each node, and that all other currents are leaving the node, applying the KCL to the 3 nodes yields

\[
\begin{align*}
(V_1 - V_2)Y_a + (V_1 - V_3)Y_b + V_1Y_d &= I_1 \\
(V_2 - V_1)Y_a + (V_2 - V_3)Y_c + V_2Y_e &= I_2 \\
(V_3 - V_1)Y_b + (V_3 - V_2)Y_c + V_3Y_f &= I_3
\end{align*}
\]
Node equations – the $Y_{bus}$ matrix

In matrix form,

$$\begin{bmatrix}
Y_a + Y_b + Y_d & -Y_a & -Y_b \\
-Y_a & Y_a + Y_c + Y_e & -Y_c \\
-Y_b & -Y_c & Y_b + Y_c + Y_f
\end{bmatrix}\begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix} = \begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix}$$

Which is an equation of the form:

$$Y_{bus} V = I$$

where $Y_{bus}$ is the bus admittance matrix of a system, which has the form:

$$Y_{bus} = \begin{bmatrix}
Y_{11} & Y_{12} & Y_{13} \\
Y_{21} & Y_{22} & Y_{23} \\
Y_{31} & Y_{32} & Y_{33}
\end{bmatrix}$$

$Y_{bus}$ has a regular form that is easy to calculate:

1) The diagonal elements $Y_{ii}$ equal the sum of all admittances connected to node $i$.
2) Other elements $Y_{ij}$ equal to the negative admittances connected to nodes $i$ and $j$.

The diagonal elements of $Y_{bus}$ are called the self-admittance or driving-point admittances of the nodes; the off-diagonal elements are called the mutual admittances or transfer admittances of the nodes.
**Y**_{bus} and **Z**_{bus} matrices of a power network

Inverting the bus admittance matrix **Y**_{bus} yields the bus impedance matrix:

\[ Z_{bus} = Y_{bus}^{-1} \]

Simple technique for constructing **Y**_{bus} is only applicable for components that are not mutually coupled. The technique applicable to mutually coupled components can be found elsewhere.

Once **Y**_{bus} is calculated, the solution

\[ V = Y_{bus}^{-1} I \]

or

\[ V = Z_{bus} I \]
Example (cont.)

The resulting admittance matrix is:

\[ Y_{bus} = \begin{bmatrix}
-j12.576 & j5.0 & 0 & j6.667 \\
 j5.0 & -j12.5 & j5.0 & j2.5 \\
 0 & j5.0 & -10.625 & j5.0 \\
 j6.667 & j2.5 & j5.0 & -j14.167 \\
\end{bmatrix} \]

The current vector for this circuit is:

\[ I = \begin{bmatrix}
1.0 \angle -80^\circ \\
0 \\
0.563 \angle -112^\circ \\
0 \\
\end{bmatrix} \]

The solution to the system of equations will be

\[ V = Y_{bus}^{-1}I = \begin{bmatrix}
0.989 \angle -0.60^\circ \\
0.981 \angle -1.58^\circ \\
0.974 \angle -2.62^\circ \\
0.982 \angle -1.48^\circ \\
\end{bmatrix} \]
Problems

• 10.3
• 10.5
• 10.7, 10.8, 10.9