1. True or False. [5 points each]
   (a) _____ A good programmer should never use linear search.
   (b) _____ If a collision occurs in a hash table, there must be some error in the implementation.
   (c) _____ The height of an AVL tree with $n$ nodes is $O(\log n)$.
   (d) _____ The height of a 2-3 tree with $n$ nodes is $O(\log n)$.
   (e) _____ A stack is an example of a search structure.
   (f) _____ An item can always be inserted into a min-heap of size $n$ in $O(\log n)$ time.
   (g) _____ A treap is an example of a priority queue.

2. Fill in the blanks (5 points each blank).
   (a) Probing is used in ________________ hashing.
   (b) A ________________ hash table is designed so that the size of the table is exactly the number of data and there are no collisions.
   (c) A 2-3 tree which holds 400 data items must have height at least _____ and at most ____. (Exact answers, please.)
   (d) A condition that is
      i. True for the first iteration of a loop, and
      ii. true at the end of any iteration of that loop, provided it is true at the beginning of that iteration,
      is called a ________________ of that loop. [2 word answer]
   (e) You need to design a hash function for a hash table that will be used for a set of data items. As the program runs, new data items will be added. Your hash function should satisfy the following three conditions.
      i. ________________
      ii. ________________
      iii. ________________
(f) For each of the following blanks, the correct answer is stack, queue, list, array, heap, or search structure.

i. pop is an operator of __________.

ii. find is an operator of __________.

iii. You would use a __________ to do breadth first search.

iv. You would use a __________ to hold the records of the customers of a business.

v. You would use an __________ to keep track of the number of times each word occurs in a Shakespeare play.

vi. You would use a __________ to match left with right parentheses in an algebraic expression.

vii. Every time you get money, you pay as many bills as possible, in order of urgency. Urgency is determined by due date, not the date that you received the bill.

You would use a __________ to store your unpaid bills.

[20 points] Build a binary search tree, starting from an empty tree, inserting the following items one at a time: Moe Abe Joe Nan Ted Kim Sam Ron Dan Sue Zed. Once an item is inserted into the tree, it is not moved.

3. [20 points] Suppose that the items of a queue are A, H, K, B, T in that order, where A is the front item.

(a) Sketch the appearance of a circular linked list implementation of that queue.

(b) Insert the item L into that queue. Show the steps. (You should draw at least two additional figures.)

4. Consider the following AVL tree T.

   B       C
  / \
 A   D
   / \ / \
  G  H I

(a) [10 points] An AVL tree should have some additional data, missing in this figure, at each node. Draw in those missing data.

(b) [10 points] List the nodes of T in preorder, inorder, postorder, and level order.

(c) [20 points] Insert F into T, and rebalance. Show the steps.
5. [30 points] Consider the 2-3 tree illustrated below. Illustrate the tree after the letter J is inserted. (You are not required to show steps, but it doesn’t hurt.)
6. [30 points] Consider the heap of 12 elements, implemented as an array as illustrated below. Illustrate the implementation after insertion of the letter E. (You are not required to show steps, but it doesn’t hurt.)

```
B M K Q N S L Z W P R U X B V W K A
12
```

7. [30 points] Consider the treap illustrated below, where the heap key is a randomly chosen integer in the range 0 . . . 99. A new item, “Fay,” is inserted, and the heap key “12” is chosen. Show the treap after that insertion. Show the intermediate steps.
8. [20 points] You are building a cuckoo hash table for the following data set consisting of 8 items. The indices of your hash table are \{0, 1, \ldots, 9\}. The two hash values for each item are listed in the first and second columns of the array below.

Walk through the steps of inserting the items, in the order given in the array.

<table>
<thead>
<tr>
<th>Name</th>
<th>1</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ann</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Dan</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>Kim</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>Sam</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>Zoe</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>Ted</td>
<td>5</td>
<td>0</td>
</tr>
<tr>
<td>Kat</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>Max</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

9. [30 points]

By drawing pictures and writing words, define the loop invariant of the partition phase of quicksort.

10. [30 points]

Walk through the steps of heapsort for the following array. Show the complete array at each step where items change position. At each step, indicate the current extent of the heap.

Z B F G P W A R