Name:__________________________________________________________

The entire examination is 345 points.

1. True or False. [5 points each]
   (a) _______ A good programmer should never use linear search.
   (b) _______ The height of an AVL tree with $n$ nodes is $O(\log n)$.
   (c) _______ The height of a 2-3 tree with $n$ nodes is $O(\log n)$.
   (d) _______ A stack is an example of a search structure.
   (e) _______ An item can always be inserted into a min-heap of size $n$ in $O(\log n)$ time.
   (g) _______ A treap is an example of a priority queue.

2. Fill in the blanks (5 points each blank).
   (a) A 2-3 tree which holds 400 data items must have height at least _______ and at most _______ (Exact answers, please.)
   (b) A condition that is
       i. True for the first iteration of a loop, and
       ii. true at the end of any iteration of that loop, provided it is true at the beginning of that iteration,
       is called a ________________ of that loop. [2 word answer]

3. For each of the following blanks, the correct answer is stack, queue, list, array, heap, or search structure.
   (a) pop is an operator of ____________ .
   (b) find is an operator of ____________ .
   (c) You would use a ____________ to do breadth first search.
   (d) You would use a ____________ to hold the records of the customers of a business.
   (e) You would use an ____________ to keep track of the number of times each word occurs in a Shakespeare play.
(f) You would use a __________ to match left with right parentheses in an algebraic expression.

(g) Every time you get money, you pay as many bills as possible, in order of urgency. Urgency is determined by due date, not the date that you received the bill.

You would use a __________ to store your unpaid bills.

4. [20 points] Build a binary search tree, starting from an empty tree, inserting the following items one at a time: Moe Abe Joe Nan Ted Kim Sam Ron Dan Sue Zed. Once an item is inserted into the tree, it is not moved.
5. [20 points] Suppose that the items of a queue are $A, H, K, B, T$ in that order, where $A$ is the front item.

(a) Sketch the appearance of a circular linked list implementation of that queue.

(b) Insert the item $L$ into that queue. Show the steps. (You should draw at least two additional figures.)
6. Consider the following AVL tree $T$.

(a) [10 points] An AVL tree should have some additional data, missing in this figure, at each node. Draw in those missing data.

(b) [10 points] List the nodes of $T$ in preorder, inorder, postorder, and level order.

(c) [20 points] Insert $F$ into $T$, and rebalance. Show the steps.
7. [30 points] Consider the 2-3 tree illustrated below. Illustrate the tree after the letter J is inserted. (You are not required to show steps, but it doesn’t hurt.)
8. [30 points] Consider the heap of 12 elements, implemented as an array as illustrated below. Illustrate the implementation after insertion of the letter E. (You are not required to show steps, but it doesn’t hurt.)

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BMKQNSLZWPUBVWKAB
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[12]
9. [30 points] Consider the treap illustrated below, where the heap key is a randomly chosen integer in the range 0...99. A new item, “Fay,” is inserted, and the heap key “12” is chosen. Show the treap after that insertion. Show the intermediate steps.
10. [30 points]

(a) Write a topological ordering of the nodes of the weighted directed graph shown in the figure below.
(b) Solve the single source shortest path problem for that graph.

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(a) Topological ordering: 

(b) Shortest path problem: 

Source: s, Target: e, Path: s → b → d → e, Distance: 14
11. [30 points]

By drawing pictures and writing words, define the loop invariant of the partition phase of quicksort.
12. [30 points]
Walk through the steps of heapsort for the following array. Show the complete array at each step where items change position. At each step, indicate the current extent of the heap.

Z B F G P W A R