1. True or False. [5 points each]
   (a) ______ A good programmer should never use linear search.
   (b) ______ The height of a binary tree with \( n \) nodes is \( O(\log n) \).
   (c) ______ \( 3^n = O(2^n) \).
   (d) ______ The height of a 2-3 tree with \( n \) nodes is \( O(\log n) \).
   (e) ______ \( \log \log n = O(\log n) \).
   (f) ______ A stack is an example of a search structure.
   (g) ______ \( n^2 = O(n \log n) \).
   (h) ______ An item can always be inserted into a min-heap of size \( n \) in \( O(\log n) \) time.
   (i) ______ A treap is an example of a priority queue.

2. Fill in the blanks (5 points each blank).
   (a) A 2-3 tree which holds 400 data items must have height at least ______ and at most ______ (Exact answers, please.)
   (b) A condition that is
      i. True for the first iteration of a loop, and
      ii. true at the end of any iteration of that loop, provided it is true at the beginning of that iteration,
      is called a ____________________ of that loop. [2 word answer]
   (c) The three kinds of priority queues we have discussed in class are:
      i. ___________________________
      ii. ___________________________
      iii. ___________________________
(d) For each application, name the kind of search structure you would use.

i. The expected number of items in the structure is only two, but there is a small probability that it will be as large as ten.

ii. You will execute find many times, but all items will be inserted at once at the beginning of the program, and there will never be another insert.

(e) For each of the following blanks, the correct answer is stack, queue, list, array, heap, or search structure.

i. pop is an operator of __________.

ii. find is an operator of __________.

iii. You would use a __________ to do breadth first search.

iv. You would use a __________ to hold the records of the customers of a business.

v. You would use an __________ to keep track of the number of times each student in a class asks a question.

vi. You would use a __________ to match left with right parentheses in an algebraic expression.

vii. Every time you get money, you pay as many bills as possible, in order of urgency. Urgency is determined by due date, not the date that you received the bill. You would use a __________ to store your unpaid bills.

3. [20 points] Build a binary search tree, starting from an empty tree, inserting the following items one at a time: Moe Abe Joe Nan Ted Kim Sam Ron Dan Sue Zed. Once an item is inserted into the tree, it is not moved.
4. State the asymptotic complexity of each of the following code fragments.
5. By drawing pictures and writing words, define the loop invariant of the partition phase of quicksort.

6. [15 points] Describe the simple form of heapsort that we discussed in class.

7. [15 points] Describe polyphase mergesort.

8. [15 points] Describe binary tree sort.
9. [20 points] Suppose that the items of a queue are \( A, H, K, B, T \) in that order, where \( A \) is the front item.

(a) Sketch the appearance of a circular linked list implementation of that queue.

(b) Insert the item \( L \) into that queue. Show the steps. (You should draw at least two additional figures.)

10. [20 points] Consider the following binary tree \( T \).

List the nodes of \( T \) in preorder, inorder, postorder, and level order.
11. [30 points] Consider the 2-3 tree illustrated below. Illustrate the tree after the letter J is inserted. (You are not required to show steps, but it doesn’t hurt.)

```
  G   Q
/ \   / \  
C   E   L   N  
|   |   |   |   |
A   C   E   G   L   N
```

12. [30 points] Consider the heap of 12 elements, implemented as an array as illustrated below. Illustrate the implementation after insertion of the letter E. (You are not required to show steps, but it doesn’t hurt.)

```
B M K Q N S L Z W P R U X B V W K A
```

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13. [30 points] Consider the treap illustrated below, where the heap key is a randomly chosen integer in the range 0…99. A new item, “Fay,” is inserted, and the heap key “12” is chosen. Show the treap after that insertion. Show the intermediate steps.
14. [30 points]

(a) Write a topological ordering of the nodes of the weighted directed graph shown in the figure below.
(b) Solve the single source shortest path problem for that graph.