Q1. Solve the following recurrences.
   a) \( F(n) = F(n/3) + 9 \)
   
   A:
   
   b) \( G(n) = 8G(n/2) + n^3 \)
   
   A:
   
   c) \( H(n) = H(n/2) + n^2 \)
   
   A:
   
   d) \( T(n) = T(\sqrt{n}) + 1 \)
   
   A:
   
   e) \( F(n) = F(n-2) + 5n^2 \)
   
   A:

Q2. Determine the asymptotic time complexity, in terms of \( n \), of each of these code fragments.

   a) 
   
   for(int i = 1; i < n; i = i*2)
   
   for(int j = 1; j < i; j++)
   
   cout << "Hello world" << endl;

   A:

   b) 
   
   for(int i = 1; i < n; I = i*2)
   
   for(int j = 1; j < n; j = j++)
   
   cout << "Hello world" << endl;

   A:

   c) 
   
   for(int i = n; i > 1; i = sqrt(i))
   
   cout << "Hello world" << endl;

   A:
Q3. Consider the following procedure (it is not written in any particular programming language).

```plaintext
procedure q(int n) {
    if (n > 0) {
        int m = n/2;
        q(m);
        q(m-1);
        q(m-2);
        q(m-3);
    }
}
```

You need to analyze the time complexity of this procedure, in terms of the parameter n.

(a) What is the asymptotic time complexity of this procedure, in terms of the parameter n? Give an argument to support your answer.

A:

(b) Write code for this procedure, in any programming language you wish. Run the procedure for various values of n, including 10, 100, 1000, and 10000. Turn in a graph summarizing the results of your experiment, and compare your results to your answer in part (a).

A:
Q4. This problem deals with Euclid's algorithm for finding the greatest common divisor of two non-negative integers. Suppose that $n$ and $m$ are two positive integers, and that $m < n$. How many steps does it take Euclid's algorithm to compute the greatest common divisor of $n$ and $m$? I want the answer expressed asymptotically in terms of $n$, for example, $O(n)$ or $\Theta(n)$, or $O(\log n)$, or whatever.

You may assume that it takes one time step to do any operation, such as $\text{mod}$, or to do an assignment.

You must give the sharpest possible answer. If the correct answer is $\Theta(n)$ and you write $O(n)$, for example, you will lose points, even though technically your answer is correct, because it is not the sharpest possible answer.

Do not give a proof that your answer is correct.