1. True or False. [5 points each]

(a) _____ Computers are so fast today that complexity theory is only of theoretical, but not practical, interest.
(b) _____ If any problem can be precisely formulated in a mathematical way, there is an algorithm that solves it.

2. Fill in the blanks. [5 points each blank.]

(a) In hash tables, the most popular way to solve collision is by __________.
(b) What is the only difference between the abstract data types queue and stack?

(c) Name a well-known divide-and-conquer searching algorithms.

3. Solve each of the following recurrences, giving the answer in terms of O, Θ, or Ω, whichever is most appropriate [10 points each].

(a) $T(n) < T(n - 2) + n^2$
(b) \( F(n) \geq F(\sqrt{n}) + \lg n \)

(c) \( G(n) \geq G(n - 1) + \lg n \)

(d) \( F(n) = 4F(n/2) + n^2 \).

(e) \( H(n) \leq 2H(\sqrt{n}) + O(\log n) \).
(f) $K(n) = K(n - \sqrt{n}) + 1.$

(g) $F(n) = 4F\left(\frac{3n}{4}\right) + n^5$ (No, you don’t need a calculator.)

4. [15 points] Consider the following procedure:

```cpp
void george(int n)
{
    int m = n;
    while (m > 1)
    {
        for (int i = 1; i < m; i++)
            cout << "I cannot tell a lie. I chopped down the cherry tree."
                 << endl;
        m = m/2;
    }
}
```

Consider the question of how many lines of output the execution of `george(n)` would produce. Write down an appropriate recurrence for this question, and give an asymptotic solution in terms of $n$, using either $O$, $\Omega$, or $\Theta$, whichever is most appropriate.
5. Give a **mathematically correct** definition of the statement, “\( f(n) = \Omega(n^3) \).” (If you write more than 15 words, your answer is probably wrong. I will take off points if you give an example, or write anything else that is unnecessary.) [15 points]

6. Show that \( n = \Omega(\log n) \). Write down the complete proof.

7. Show that \( \sqrt{n} = O(n) \). Write down the complete proof.