1. Let $G$ be the weighted directed graph represented below as an array of in-neighbor lists. For example, there is an edge from 2 to 3 of weight -2.

- $1: (3, 2)$
- $2: (1, 1), (5, 4)$
- $3: (1, 0), (2, -2), (5, -2)$
- $4: (3, 1)$
- $5: (3, 4), (4, 2), (7, 3)$
- $6: (2, 10), (5, 3)$
- $7: (4, 2), (6, 1)$

(a) Compute the array of out-neighbor lists of $G$.

(b) Execute the Bellman-Ford algorithm for the single source shortest path problem on $G$, where the source is the vertex 1. Show the two arrays after each iteration of the outer loop.
2. Describe an algorithm for finding the length of the longest strictly monotone increasing subsequence of a sequence of \( n \) numbers. (There may be more than one equally long longest subsequence.) For example if the original sequence is 2, 5, 0, 1, 6, the answer is 3, and there are two different monotone increasing subsequences of length 3.

Be sure to explain the time complexity of your algorithm. The most obvious algorithm takes \( O(n^2) \) time. However, there is an algorithm whose time complexity is \( O(n \log n) \).
3. A word ladder in a particular language is a sequence of words of that language such that each word differs from its predecessor in one place. For example, WARM, WARD, CARD, CORD, COLD is a word ladder in English. The problem of finding the shortest word ladder of 4-letter words in English is simply the single pair shortest path problem in a rather large graph.

The A* algorithm can be used to find the shortest ladder between two words. Use that algorithm to find the shortest word ladder in English between JOIN and BULL. As your heuristic, use Hamming distance. The Hamming distance between two words of the same length is the number of positions at which the symbols differ. For example, the Hamming distance between JOIN and BULL is 4, while the Hamming distance between BARN and BAND is 2.

In order to execute your algorithm, you need a list of four letter words in English. You can use the list at: