Clues for the Underemployed Watchman

Clues for the Oblivious Forgetful Adversary

As we know, there is no online algorithm for computing the optimal service of a request sequence for the Underemployed Watchman problem. However, there is an online algorithm for computing the optimal cost of a request sequence.\(^1\)

Each position (left or right) is marked in one of three ways: Y(es), N(o), or M(aybe). Initially, the left is marked Y, the right is marked N, and \(cost_{opt} = 0\).

Mark the left side Y and the right side N.
Let \(cost_{opt} = 0\).
For each request, execute the following:

<table>
<thead>
<tr>
<th>Condition</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>If the requested side is marked Y, do nothing.</td>
<td></td>
</tr>
<tr>
<td>Else if the requested side is marked M, mark</td>
<td>that side Y and the other side N.</td>
</tr>
<tr>
<td>Else if the requested side is marked N, mark</td>
<td>both sides M and increment (cost_{opt}) by 1.</td>
</tr>
</tbody>
</table>

The final value of \(cost_{opt}\) is the optimum cost of serving the sequence.

Let \(p^t\) be the probability that the marks are MM after \(t\) requests, if the request sequence is generated by the oblivious forgetful adversary. Then \(p^0 = 0\), and \(p^t = \frac{1}{2}(1 - p^{t-1})\) for all \(t > 0\). (Do you see why)? Thus, you can compute \(p^t\) for any \(t \geq 0\), although the fractions get more and more complex as \(t\) increases.

However, the sequence \(\{p^t\}\) converges to a simple fraction. What is that fraction? You can then compute the limiting value of the average expected optimum cost, namely:

\[
\lim_{n \to \infty} \frac{E(cost_{opt}^n)}{n}
\]

---

\(^1\)The same is true for the paging problem.